CHAPTER 2

Basic Principles of Shock Compression

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2.1. Shock-Wave Concept

Shock waves are the ubiquitous result of matter moving at velocities faster than the speed at which adjacent material can move out of the way. Examples range in scale from the shock waves generated by the collapse of microscopic cavitation bubbles to light-year scale "collisionless shocks" in the interstellar medium. The concept of a shock wave is well illustrated by the flow of snow in front of a moving snowplow (Fig. 2.1). When a plow begins moving into fresh, loose snow, a layer of packed snow builds up on the blade. The interface between the fresh snow and packed snow moves out ahead of the blade at a speed greater than that of the plow.

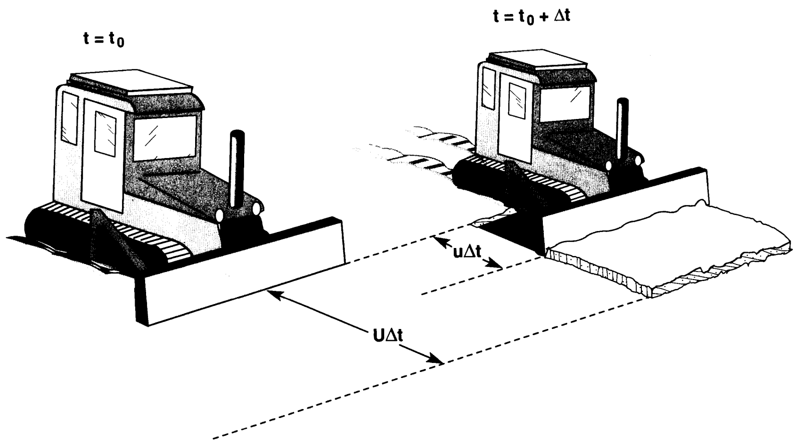

Figure 2.1. Conceptual depiction of a shock wave as a discontinuity between undisturbed snow and packed snow. At time $t = t_0$, the plow begins to move at velocity u. At $t = t_0 + \Delta t$, the plow has moved a distance $u\,\Delta t$, but the discontinuity between loose and packed snow has moved a distance $U\,\Delta t$, where U is the velocity of the front of the packed snow.

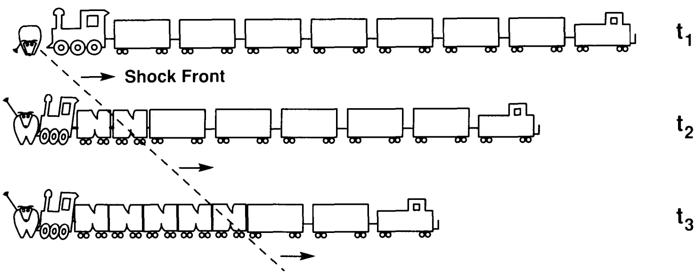

Figure 2.2. Shock wave in a freight train resulting from impact between the cowcatcher and bull. In this case, the bull is infinitely massive.

Perhaps a more graphic illustration of a shock wave is depicted in Fig. 2.2, which shows what happens when a bull is struck by a moving train. The shock wave in this example is a moving discontinuity that separates undisturbed boxcars from crushed boxcars, and moves from left to right away from the point of collision. In this case, the frame of reference is such that the shock wave is stopping the medium (freight cars) into which it is moving. We have taken the liberty of assuming that the bull is infinitely massive and is not accelerated by the collision.

In this chapter, we will discuss the basic properties of shock waves, and the concepts that allow determination of the response of materials to intense, short-duration loading. We start by considering the properties of shock waves that make them unique. The properties that distinguish shock loading from other forms of stress loading on materials are the highly transient nature and rapid rates associated with the application of stress. These rates exceed those that can be achieved by any other method of subjecting materials to stress loading.

The ideas developed in this chapter are descriptive of shock waves in fluids. Solids have many significant features that distinguish them from liquids and gases, such as shear strength, polymorphic phase transformations, heterogeneous structure, anisotropy, and viscoplastic behavior. The influences of these special properties of solids on shock compression are the topics of several of the other chapters, and for the most part are ignored in this introduction to the basic principles of shock compression.

2.2. Conservation Equations

Because mass, momentum, and energy are conserved across the shock discontinuity, we can write conservation equations. To illustrate the conservation of mass, we again allude to a snowplow moving into fresh snow. For simplicity, we can assume that there is no lateral motion, so that all flow is uniaxial (i.e., along the direction the plow is moving). This situation is shown in Fig. 2.1. At time t_0 the plow begins moving at velocity u from its starting point. At some later time, $t_0 + \Delta t$, the plow has moved a distance $u \, \Delta t$ from its initial

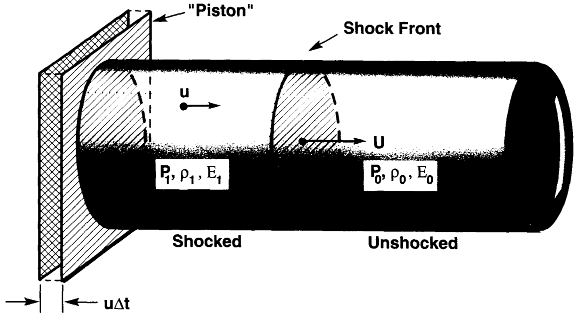

Figure 2.3. A rigid piston drives a shock wave into compressible fluid in an imaginary flow tube with unit cross-sectional area. The shock wave moves at velocity U into fluid with initial state "0," which changes discontinuously to state "1" behind the shock wave. Particle velocity u is identical to the piston velocity.

position. Since the discontinuity between loose and packed snow has advanced ahead of the plow, its velocity U must be greater than u, and it travels a distance $U\ \Delta t$ during the same time interval. If the mass of fresh snow per unit area of the street is ρ_0, the amount that is encompassed by the discontinuity has a mass of $L\rho_0 U\ \Delta t$, where L is the length of the plow blade. Once the snow is packed, its mass per unit area increases to ρ. The mass of snow between the blade and the discontinuity is $L\rho(U - u)\ \Delta t$. Because mass is conserved across the discontinuity, these values are equal. Dividing both quantities by L and equating yields

$$\rho_0 U = \rho(U - u) \qquad \text{(conservation of mass).} \tag{2.1}$$

This equation was derived for a two-dimensional system, where the areal density, ρ, of the snow was used. It applies equally to a three-dimensional system, where the discontinuity is a plane instead of a line, and ρ is the volume density.

To permit a more general discussion, we can replace the snowplow with a piston, and replace the snow with any fluid (Fig. 2.3). We consider the example shown in a reference frame in which the undisturbed fluid has zero velocity. When the piston moves, it applies a planar stress, σ_x, to the fluid. For a non-viscous, hydrodynamic fluid, the stress is numerically equal to the pressure, P.[1] The pressure induces a shock discontinuity, denoted by $\mathscr{S}$, which propagates through the fluid with velocity U. The velocity u of the piston, and the shocked material carried with it (with respect to the stationary frame of reference), is called the *particle velocity*, since that would be the velocity of a particle caught up in the flow, or of a "particle" of the fluid.

The importance of the distinction between shock velocity and particle velocity cannot be overemphasized. The particle velocity refers to the velocity

[1] The words "stress" and "pressure" are used interchangeably for fluids considered in this chapter, but the distinction is important and will be made in later chapters.

that a given element in the material acquires, as a result of the shock wave passing over the element. The shock velocity is the velocity with which the disturbance moves through the body. The shock velocity is always greater than the particle velocity[2] and typically a few times larger at low stresses (for which the fluid experiences only small compression). A good example of the difference is illustrated by debris, such as dust or paper, blown by a gust of wind. The velocity (analogous to shock velocity) of the gust is observed by how fast the gust travels over the ground. The particle velocity is observed by the velocity of the dust or paper accelerated by the moving gust of wind.[3] A "20-mph wind" refers to the particle velocity of the moving air. The front can move across an area much faster. Note that in the example of the snow plow, the shock velocity is greater than the particle velocity, which is identical to the velocity of the plow.

Returning to Fig. 2.3, the conservation of momentum is expressed by considering the forces within an imaginary tube of unit area oriented along the direction of flow, with one end in the undisturbed layer and the other end in the compressed layer. The force acting from the left side on the fluid within the tube is equal to the pressure P applied by the piston, while the force on the right is P_0, the pre-existing pressure. The net force acting on the system is $P - P_0$ to the right. This force must be equated to the momentum transferred from the surroundings to the fluid within the tube per unit time. The discontinuity (or shock wave) accelerates a mass equal to $\rho_0 U$ per unit time to the velocity u, which results in a momentum transfer of $\rho_0 Uu$ of the mass element. Equating these two quantities results in the relation

$$P - P_0 = \rho_0 Uu \qquad \text{(conservation of momentum).} \qquad (2.2)$$

The equation that expresses conservation of energy can also be determined by considering Fig. 2.3. Since the piston moves a distance $u\,\Delta t$, the work done by the piston on the fluid during this time interval is $Pu\,\Delta t$. The mass of material accelerated by the shock wave to a velocity u is $\rho_0 U\,\Delta t$. The kinetic energy acquired by this mass element is therefore $(\rho_0 Uu^2)\,\Delta t/2$. If the specific internal energies of the undisturbed and shocked material are denoted by E_0 and E, respectively, the increase in internal energy is $(E - E_0)\rho_0 U\,\Delta t$ per unit mass. The work performed on the system is equal to the sum of kinetic and

[2] This statement refers to a frame of reference in which the initial state of the material is stationary. It is not true for all frames of reference, e.g., one which moves with the shock velocity.

[3] The first estimate of the yield of the first atomic bomb was made by such a simple measurement. In the moments following detonation of the device in 1945, at the Trinity site on the Alamogordo Bombing Range (now White Sands Missle Range), New Mexico, Enrico Fermi performed an experiment. He dropped bits of paper and measured the distance they traveled before they hit the ground. Knowing how long it took them to fall, this distance gave a measure of particle velocity. From this he was able to determine the strength of the shock wave and estimate the yield of the bomb.

internal energy, which yields the equation,

$$Pu = \tfrac{1}{2}(\rho_0 U u^2) + \rho_0 U(E - E_0) \qquad \text{(conservation of energy).} \qquad (2.3)$$

These equations can be combined to eliminate the velocities, yielding the *Rankine–Hugoniot equation* for internal energy jump in terms of pressures and specific volumes ($V \equiv 1/\rho$)

$$E - E_0 = \tfrac{1}{2}(V_0 - V)(P + P_0) \qquad \text{(Rankine–Hugoniot equation).} \qquad (2.4)$$

To reiterate, the development of these relations, (2.1)–(2.3), expresses conservation of mass, momentum, and energy across a planar shock discontinuity between an initial and a final *uniform* state. They are frequently called the "*jump conditions*" because the initial values jump to the final values as the idealized shock wave passes by. It should be pointed out that the assumption of a "discontinuity" was not required to derive them. They are equally valid for any "steady" compression wave, connecting two uniform states, whose profile does not change with time. It is important to note that the initial and final states achieved through the shock transition must be states of mechanical equilibrium for these relations to be valid. The time required to reach such equilibrium is arbitrary, providing the transition wave is steady. For a more rigorous discussion of steady compression waves, see Courant and Friedrichs (1948).

The jump conditions are frequently applied with a high degree of accuracy to "unsteady" shock waves for which the state behind the shock front is changing with time. Such unsteady behavior can be caused by a decompression wave overtaking the shock front (see Section 2.8), or by rate-dependent properties of the medium such as viscoelasticity or chemical reactions. As long as the state of the material changes slowly, relative to the rate of change at the shock front (i.e., near-equilibrium conditions are achieved), the jump conditions are a valid and useful approximation. In addition, it can be shown that, even though these equations were derived for a planar shock front, for generalized nonplanar (curvilinear) shocks they are still valid locally (see, e.g., Owczarek, 1964).

For shock waves in solids, the shock pressure P is typically much greater than the initial pressure P_0, which is normally ambient atmospheric conditions, so that P_0 is usually neglected. E_0 can also be taken to be zero, since internal energy is a thermodynamic state function and can be referenced to any initial state. Removing E_0 and P_0 from the jump conditions results in their "common" form

$$\rho_0/\rho = 1 - u/U, \qquad (2.5)$$

$$P = \rho_0 U u, \qquad (2.6)$$

$$E = (P/2)(V_0 - V). \qquad (2.7)$$

There will be many circumstances under which the jump conditions must be applied to material that is already in motion, perhaps from the passage of an earlier shock wave. If the general state of the material, into which the shock

Table 2.1. Jump conditions in Eulerian coordinates.

Conserved quantity	Common	General
Mass	$\rho_0/\rho = 1 - u/U$	$\rho_1/\rho = 1-(u-u_1)/(U-u_1)$
Momentum	$P = \rho_0 U u$	$P - P_1 = \rho_1(u-u_1)(U-u_1)$
Energy	$E = (P/2)(V_0 - V)$	$E - E_1 = \frac{1}{2}(P + P_1)(V_1 - V)$

under consideration is moving, is defined as state "1," the shock transition places it in a new state referred to as state "2." The jump conditions can be cast in a slightly more general form because it is not always convenient to use a reference frame that moves with the unshocked state. The "general" jump conditions are as follows, where u_1 refers to the particle velocity of the unshocked material with respect to an arbitrary reference frame

$$\rho_1/\rho = 1 - (u - u_1)/(U - u_1), \tag{2.8}$$

$$P - P_1 = \rho_1(U - u_1)(u - u_1), \tag{2.9}$$

$$E - E_1 = \tfrac{1}{2}(P + P_1)(V_1 - V). \tag{2.10}$$

The various forms of the jump conditions are summarized in Table 2.1.

The jump conditions must be satisfied by a steady compression wave, but cannot be used by themselves to predict the behavior of a specific material under shock loading. For that, another equation is needed to independently relate pressure (more generally, the normal stress) to the density (or strain). This equation is a property of the material itself, and every material has its own unique description. When the material behind the shock wave is a uniform, equilibrium state, the equation that is used is the material's thermodynamic *equation of state.* A more general expression, which can include time-dependent and nonequilibrium behavior, is called the *constitutive equation.*

2.3. The "Beads on a Wire" Model

The basic concepts of shock and particle velocities are well illustrated by an example first introduced by Duvall and Band (1968). Here we assume that a string of beads of diameter *d*, mass *m*, and spaced a fixed distance ℓ apart on a smooth (frictionless) wire is impacted by a rigid, massive piston at velocity v. Each bead is assumed to undergo perfectly elastic, rigid-body motion upon impact with its neighbor.

After impact the first bead assumes a velocity $2v$, due to its rigid elastic response. This is the instantaneous "particle velocity" that the bead acquires. The first bead travels across the gap *d* and impacts the second bead. The only way by which momentum and energy can be simultaneously conserved is for the first bead to come to rest at the instant the second bead acquires a velocity

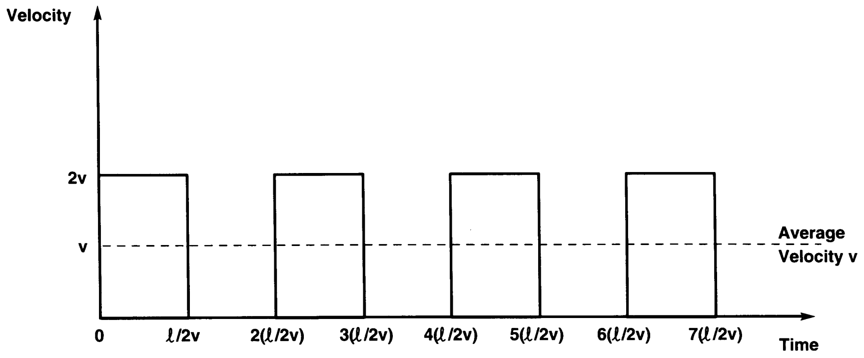

Figure 2.4. Velocity history of the first "bead on a wire." Average velocity v is identified with "drift" or with the particle velocity of the bead.

$2v$. This process proceeds sequentially down the wire. The time between collisions is defined to be τ, which is the separation distance divided by the bead velocity, $2v$. The first bead remains at rest until impacted again by the piston. It accelerates to velocity $2v$, and again collides with the second bead (now at rest), again giving up its momentum and kinetic energy. The velocity history of the first bead is illustrated in Fig. 2.4. Its velocity alternates between zero and $2v$ at equal intervals, so its mean (or drift) velocity is v. The same is true for all the other beads that have been disturbed.

This simple example illustrates the important kinematic properties of shock waves, particularly the concepts of particle velocity and shock velocity. The particle velocity is the average velocity acquired by the beads. In this example, it is the piston velocity, v. The shock velocity is the velocity at which the disturbance travels down the string of beads. In general, at time $n\ell/2v$, the disturbance has propagated to the nth bead. The distance the disturbance has traveled is therefore $n(d + \ell)$, and the shock velocity is

$$U = \frac{n(d + \ell)}{n\ell/2v} = 2v(d + \ell)/\ell. \tag{2.11}$$

Note that this is faster than the particle velocity $u = v$.

If the string of beads is considered to be a single entity, the part behind the disturbance has a kinetic energy per bead of

$$K = \tfrac{1}{2}(mu^2) = \tfrac{1}{2}(mv^2). \tag{2.12}$$

However the time-averaged kinetic energy of an individual bead is

$$E_{av} = \tfrac{1}{2}(\tfrac{1}{2}(m(2v)^2 + 0^2) = mv^2. \tag{2.13}$$

The difference between these two energies can be identified with an agitation energy of the beads that does not participate in translating the string of beads as a whole. This is analogous to the internal energy of a real material. This energy is

$$\mathscr{E} = \tfrac{1}{2}(mv^2). \tag{2.14}$$

After shock compression, the average separation of the beads is less than the original separation, ℓ. When the shock has advanced to the nth bead in time $n\ell/2v$, the piston has advanced $n\ell/2$. The linear density of beads is then $2m/(\ell + 2d)$, which implies that the compression ratio is

$$\rho_f/\rho_0 = 2(d + \ell)/(\ell + 2d), \tag{2.15}$$

where ρ_f and ρ_0 are the final and initial linear bead densities. From the previous definitions, the compression ratio can be written as

$$\rho_f/\rho_0 = U/(U - u). \tag{2.16}$$

Note that this is in agreement with (2.1). The simple bead model illustrates and extends the concepts of shock and particle velocity, kinetic and internal energy, and compression of materials by shock waves. It is also possible to define an analogous force equation for the bead model, which was also originally developed by Duvall and Band. It is clear that no forces exist between the beads, except at impact, for a perfectly elastic impact. It is also obvious that mass is conserved in the interaction. Since momentum must be conserved, Newton's second law of motion can be developed by assuming that an average force F acts on the driving piston as a result of repeated impacts with the beads. The increase in momentum of the system of beads is due to the acceleration of new beads on the undisturbed part of the wire. The new mass accelerated in time dt is $\rho_0 U\, dt$, which acquires an average velocity v. Thus,

$$F = \rho_0 U u. \tag{2.17}$$

It is instructive to collect the important relations here for comparison to the jump conditions derived in Section 2.4. When the bead parameters are replaced with the properties of particle and shock velocities, force and internal energy, the relations can be written as

$$\rho_0/\rho = 1 - u/U \qquad \text{(mass)}, \tag{2.18}$$

$$P = \rho_0 U u \qquad \text{(momentum)}, \tag{2.19}$$

$$E = \tfrac{1}{2}u^2. \qquad \text{(energy)}. \tag{2.20}$$

In (2.19), F has been replaced by P because force and pressure are identical for a one-dimensional system. In (2.20), $\mathscr{E}/m$ has been replaced by E, the specific internal energy (energy per unit mass). Note that all of these relations are independent of the physical nature of the system of beads and depend only on mechanical properties of the system. These equations are equivalent to (2.1)–(2.3) for the case where $P_0 = 0$. As we saw in the previous section, they are quite general and play a fundamental role in shock-compression studies.

2.4. Thermodynamic Effects of Shock Compression and the Hugoniot Curve

In an ideal fluid, the stresses are isotropic. There is no strength, so there are no shear stresses; the normal stress and lateral stresses are equal and are identical to the pressure. On the other hand, a solid with strength can support shear stresses. However, when the applied stress greatly exceeds the yield stress of a solid, its behavior can be approximated by that of a fluid because the fractional deviations from stress isotropy are small. Under these conditions, the solid is considered to be "hydrodynamic." In the absence of rate-dependent behavior such as viscous relaxation or heat conduction, the equation of state of an isotropic fluid or hydrodynamic solid can be expressed in terms of specific internal energy as a function of pressure and specific volume $E(P, V)$. A familiar equation of state is that for an ideal gas

$$E(P, V) = PV/(\gamma - 1), \tag{2.21}$$

where γ is the ratio of specific heats.

A commonly used equation of state for solids is the Mie–Grüneisen equation

$$E(P, V) = E_k(V) + V(P - P_k(V))/\Gamma(V), \tag{2.22}$$

where $E_k(V)$ is the specific internal energy on some reference isotherm or adiabat, $P_k(V)$ gives the known pressure dependence on the reference curve, and $\Gamma(V)$ is the Grüneisen parameter, which in general is a function of specific volume (or density).

Including the equation of state with the three jump conditions gives four equations with five variables: P, V, E, U, and u. Thus, there is only one independent variable, and a curve is defined in a five-dimensional variable space. This is called the "Rankine–Hugoniot curve", or simply "*Hugoniot.*" The Hugoniot can be represented in any two-dimensional plane. As we will see, the most useful representations are the P–V, P–u, and U–u planes. In reality, there is a family of Hugoniots for a given material, each centered on an initial state defined by P_0 and V_0. The Hugoniot centered on the initial state of a material at standard conditions is called the "*principal Hugoniot.*" It is important to emphasize that the P–V Hugoniot is *not* a path that is followed during compression or any special thermodynamic path, but rather the locus of all the possible end states that can be achieved behind a single shock wave passing through a material at a given initial state. It can be shown that the path followed by steady shock compression is the chord connecting the initial state and final shocked state (see Courant and Friedrichs, 1948). This is called the "*Rayleigh line.*"

The Rankine–Hugoniot curve is sometimes referred to as the "shock adiabat" (especially in the Soviet literature). This terminology reflects the fact that the shock process is so fast that there is insufficient time for heat to flow between the system and the surroundings. It is somewhat misleading, however, because the term "adiabat" often refers to a reversible path in thermodynamic space. A reversible, adiabatic change in the thermodynamic state of a material is one in which there is no increase in entropy; it is an "*isentropic*" change, but adiabatic changes are not isentropic in general. The curve that represents an isentropic change is the "isentrope," and every material has a family of isentropes, each corresponding to a different entropy. The isentrope passing through standard conditions is the "principal isentrope."

The Rankine–Hugoniot curve has frequently been called the "Hugoniot equation of state." This term is a misnomer, because the equation of state of a material is a surface in thermodynamic space (two independent variables), whereas the Hugoniot is a curve (one independent variable). On the other hand, shock-wave experiments can be used to determine the Hugoniot of a material. By making assumptions about the form of that material's equation of state, it can be constructed at high pressure from the Hugoniot data. This point will be discussed in a later chapter.

2.5. Hugoniot Differential Equation

The Hugoniot can be described with a differential equation by taking the total differential of the Rankine–Hugoniot equation (2.4)

$$dE = \tfrac{1}{2}(V_0 - V)\,dP - \tfrac{1}{2}(P + P_0)\,dV, \tag{2.23}$$

and setting it equal to the thermodynamic expression

$$dE = T\,dS - P\,dV. \tag{2.24}$$

This yields the expression

$$T\,dS = \tfrac{1}{2}(V_0 - V)\,dP + \tfrac{1}{2}(P - P_0)\,dV. \tag{2.25}$$

Since this equation came directly from differentiation of the Rankine–Hugoniot equation, it only holds true on the Hugoniot. We can also write $T\,dS$ as a total differential in terms of dP and dV

$$T\,dS = \frac{V}{\Gamma}\,dP + \frac{K_S}{\Gamma}\,dV, \tag{2.26}$$

where $\Gamma \equiv V(\partial P/\partial E)|_V$, the thermodynamic Grüneisen parameter, and $K_S \equiv V(\partial P/\partial V)|_S$, the adiabatic bulk modulus. Equation (2.26) is a general thermodynamic equation, and unlike (2.25) its use is not restricted to the Hugoniot.

Eliminating $T\,dS$ and rearranging results in a differential equation for the Hugoniot

$$\frac{dP}{dV} = \frac{\partial P/\partial V|_S + (\Gamma/2V)(P - P_0)}{1 - (\Gamma/2V)(V_0 - V)}. \tag{2.27}$$

For a material with a known equation of state, this equation can be used to calculate the Hugoniot. Frequently, however, the Hugoniot is determined for a material with an unknown equation of state. When this is the case, (2.27) can be used with experimental data to help constrain the equation of state. By taking the limit of a small shock ($V \to V_0$, $P \to P_0$), it is clear from (2.27) that the principal isentrope is tangent to the principal Hugoniot at the initial state. Thus the isentrope is a good approximation to the Hugoniot for weak shocks. This point will be discussed further in Section 2.16.

2.6. Graphical Representations and the Rayleigh Line

The jump conditions (2.1)–(2.2) can be combined and solved for the shock and particle velocities

$$U = V_0\sqrt{(P - P_0)/(V_0 - V)}, \tag{2.28}$$

$$u = \sqrt{(P - P_0)(V_0 - V)}. \tag{2.29}$$

By considering a P–V Hugoniot in Fig. 2.5, the physical meaning of these equations becomes apparent. For a shock of given strength we can graphically connect the initial and final states with a chord, called the "Rayleigh line ($\mathscr{R}$)." From (2.28), it is clear that the slope of this line is equal to $(U/V_0)^2$. For a material that has a Hugoniot that is everywhere concave upward, it is easy to see that the shock velocity is an increasing function of shock pressure. The changes in strain and temperature across the shock front are so rapid that dissipative mechanisms, such as viscosity and heat transport, come into play in such a way that the loading path is along the Rayleigh line. These dissipa-

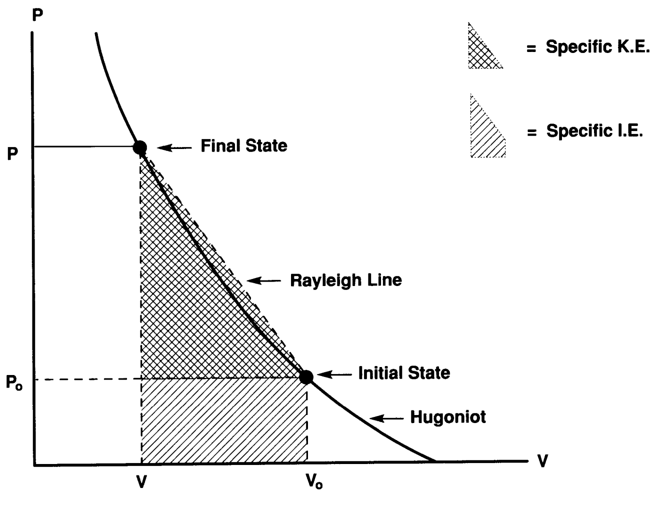

Figure 2.5. Relationship of the P–V Hugoniot to the Rayleigh line and a graphical illustration of kinetic and internal energy increase.

tive mechanisms give rise to another component of stress in addition to that due to the equation of state of the material, so the Rayleigh line must lie above the Hugoniot which defines the state behind the shock wave in the P–V plane. Hugoniots with kinks or slope discontinuities due to transitions from elastic to plastic behavior or phase transformations can thus have regions for which the initial state cannot be connected to the final state with a single Rayleigh line. In such cases, the final state is reached by a series of two or more Rayleigh lines, each with a slope in the P–V plane that is proportional to the square of the velocity of the corresponding shock wave. The criteria for shock stability, given in the next section, are not met everywhere for a material in which multiple shock waves form.

According to (2.29), the specific kinetic energy of the material behind the shock front is equal to $(P - P_0)(V_0 - V)/2$. Referring again to Fig. 2.5, one can see that this is equal to the area of the triangle bounded by the Rayleigh line, V, and P_0. The gain in specific internal energy is given by (2.4), and is equal to the area of the trapezoid bounded by the Rayleigh line and the V axis. Under the special initial condition of $P_0 = 0$ (which is often a good approximation), the kinetic energy and the internal energy generated by the shock wave are equal.

2.7. Shock Stability

In most solids, the sound speed is an increasing function of pressure, and it is that property that causes a compression wave to steepen into a shock. The situation is similar to a shallow water wave, whose velocity increases with depth. As the wave approaches shore, a small wavelet on the trailing, deeper part of the wave moves faster, and eventually overtakes similar disturbances on the front part of the wave. Eventually, the water wave becomes gravitationally unstable and overturns.

For a shock wave in a solid, the analogous picture is shown schematically in Fig. 2.6(a). Consider a compression wave on which there are two small compressional disturbances, one ahead of the other. The first wavelet moves with respect to its surroundings at the local sound speed of a_1, which depends on the pressure at that point. Since the medium through which it is propagating is moving with respect to stationary coordinates at a particle velocity u_1, the actual speed of the disturbance in the laboratory reference frame is $a_1 + u_1$. Similarly, the second disturbance advances at $a_2 + u_2$. Thus the second wavelet overtakes the first, since both sound speed and particle velocity increase with pressure. Just as a shallow water wave steepens, so does the shock. Unlike the surf, a shock wave is not subject to gravitational instabilities, so there is no way for it to overturn.

The shock wave is subject to other dissipative effects, however, such as viscosity and heat transport. It is these dissipative mechanisms that are responsible for preventing the shock from becoming a true, infinitesimally thin discontinuity. In reality, the velocity gradient can only increase until

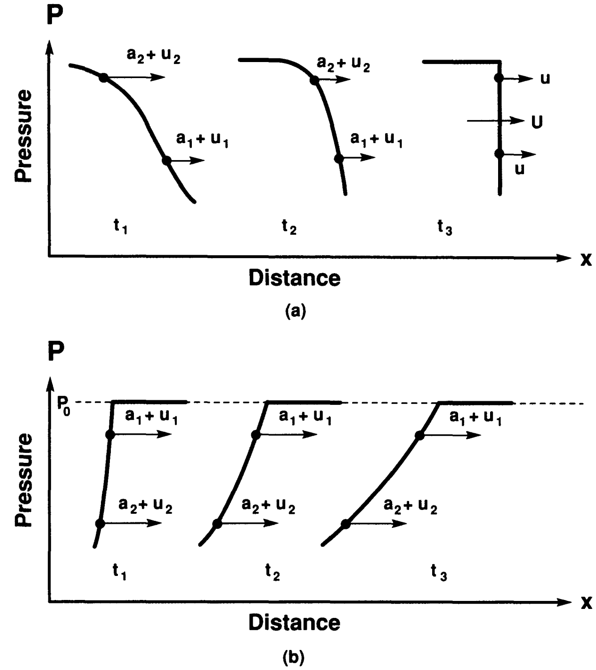

Figure 2.6. (a) Compression wave steepens to a shock wave in a medium for which stability criteria are satisfied, where the trailing part of the wave overtakes the leading part. (b) Expansion wave broadens as the leading part of the wave outruns the trailing part.

viscous stresses become important. The temperature gradient is similarly limited by thermal transport mechanisms. When these dissipative forces become significant, they begin to cancel out the effect of increasing sound speed with pressure, which is the driving force behind the steepening compression wave. Eventually, when the opposing effects cancel one another, the wave profile no longer evolves with time; it becomes a steady shock (sometimes referred to as a steady stress wave). Since nonlinear and dissipative effects only cancel when the strain rates become very large, shock wave risetimes are usually shorter than can be experimentally resolved for shock amplitudes greater than about 10 GPa for most materials. It has been shown empirically for many materials that strain rate $\dot{\varepsilon}$ varies as the stress amplitude approximately to the fourth power (e.g., Swegle and Grady, 1985). It is for this reason that high-amplitude shock waves can be considered to be discontinuities for many purposes. However, for steady waves of amplitude less than about 5 GPa, the wave risetime can be quite large (typically tens to hundreds of nanoseconds).

Since a compressional disturbance moves at the speed $a + u$, the sum of the sound speed and the particle velocity at the point through which the disturbance is passing, a disturbance trailing the shock wave cannot be slower than the shock velocity. Otherwise, it would not be able to catch up; the shock wave would be unstable with respect to small trailing perturbations and would decay. Similarly, a small compressive disturbance ahead of the shock wave must be moving slower than the shock front. Otherwise, it would outrun the shock wave, and the compression wave would not be steady. Thus, the conditions for stability are

$$\frac{da}{dP} > 0 \quad \text{(sound speed increases with pressure)}, \tag{2.30}$$

$$a + u \geq U \quad \text{(shock wave is subsonic w.r.t. shocked state)}, \tag{2.31}$$

$$U > a_0 \quad \text{(shock wave is supersonic w.r.t. unshocked state)}, \tag{2.32}$$

where a_0 is the sound speed of the initial state.

The adiabatic sound speed of a hydrodynamic medium is

$$a = V\sqrt{(-\partial P/\partial V)_s}. \tag{2.33}$$

Substituting (2.33) and (2.28) into the stability condition (2.32) yields the inequality

$$(P - P_0)/(V_0 - V) > -(\delta P/\delta V)_{s,0}. \tag{2.34}$$

The left-hand side of the inequality is the slope of the Rayleigh line, and the right-hand side is the slope of the isentrope centered on the initial state. We showed in Section 2.5 that the isentrope and Hugoniot are tangent at the initial state. Thus, the stability condition which requires that the shock wave be supersonic with respect to the material ahead of it is equivalent to the statement that the Rayleigh line must be steeper than the Hugoniot at the initial state.

With similar substitutions, the stability condition (2.31) can be expressed as the inequality

$$(P - P_0)/(V_0 - V) \leq -(\delta P/\delta V)_s, \tag{2.35}$$

where the right-hand side is now the slope of the isentrope at the shock state. Substituting equation (2.27) into (2.35) gives the inequality

$$(P - P_0)/(V_0 - V) \leq -(dP/dV)_H(1 - (V_0 - V)\Gamma/2V) - (P - P_0)\Gamma/2V, \tag{2.36}$$

which, with some rearranging reduces to

$$(P - P_0)/(V_0 - V) \leq -(dP/dV)_H. \tag{2.37}$$

The stability condition that the shock wave is subsonic with respect to the shocked material behind it is equivalent to the statement that the Hugoniot must be steeper than the Rayleigh line at the final state.

It should be noted that not all materials satisfy these stability criteria. For example, over a range of low pressures, the sound speed of fused silica decreases with pressure, so shock waves cannot be supported. As pointed out in the previous section, other solids exhibit elastic–plastic behavior, or undergo polymorphic phase transformations, in which the sound speed decreases discontinuously with shock pressure. At pressures slightly above these transitions, shock waves will split into two or more wavefronts. Between these wavefronts, the stability criteria are not fulfilled, but within each one they are, so that the individual shock waves are stable.

2.8. Expansion Waves

Expansion waves are the mechanism by which a material returns to ambient pressure. In the same spirit as Fig. 2.2, a rarefaction is depicted for intuitive appeal in Fig. 2.7. In this case, the bull has a finite mass, and is free to be accelerated by the collision, leading to a free surface. Any finite body containing material at high pressure also has free surfaces, or zero-stress boundaries, which through wave motion must eventually come into equilibrium with the interior. Expansion waves are also known as *rarefaction waves, unloading waves, decompression waves, relief waves,* and *release waves.* Material flow is in the same direction as the pressure gradient, which is opposite to the direction of wave propagation.

In materials that support shock waves, the sound speed increases with pressure. It is this same property that causes rarefactions to spread out as they progress. In Fig 2.6(b), an unloading wave is shown propagating into a stationary material with some initial pressure P_0. This time, we consider the evolution of two small decompressional disturbances. The first disturbance moves at the local sound speed of a_1 into its surroundings, which have begun

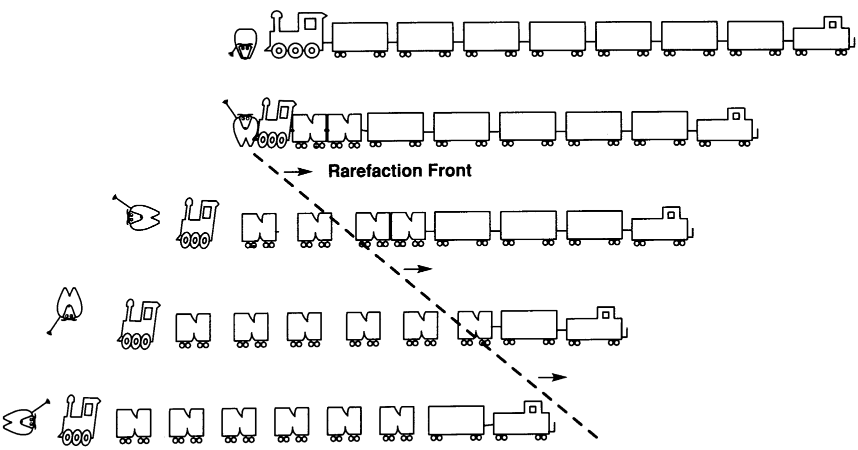

Figure 2.7. When a freight train impacts a bull with finite mass, the resulting shock wave is quickly followed by a rarefaction wave, which overtakes and decays the original shock wave.

to be accelerated in the other direction by the expansion wave. With respect to a laboratory frame of reference, the velocity of this expansion wavelet is thus $a_1 + u_1$, where these velocities are in opposite directions. As before, the second disturbance moves inward at a net velocity of $a_2 + u_2$. Because of the sound speed dependence on pressure, a_2 is less than a_1. In addition, the material at point 2 has been accelerating longer than that at point 1, so its velocity u_2 is faster (in the negative direction) than u_1. For any pair of rarefaction disturbances on an expansion wave, $(a_2 + u_2) < (a_1 + u_1)$ when $P_2 < P_1$. This results in a spreading of the rarefaction zone with time.

The fact that shock waves continue to steepen until dissipative mechanisms take over means that entropy is generated by the conversion of mechanical energy to heat, so the process is irreversible. By contrast, in a fluid, rarefactions do not usually involve significant energy dissipation, so they can be regarded as reversible, or isentropic, processes. There are circumstances, however, such as in materials with elastic-plastic response, in which plastic deformation during the release process dissipates energy in an irreversible fashion, and the expansion wave is therefore not isentropic.

2.9. *x–t* Diagrams

Throughout this book, a shock pulse (a steady compression wave followed by an expansion wave) will be represented as a profile, such as in Fig. 2.6. In Fig. 2.8 we show a series of "P–x snapshots" of pressure versus propagation distance x for an initially square pulse, at a series of times t. For a fluid with

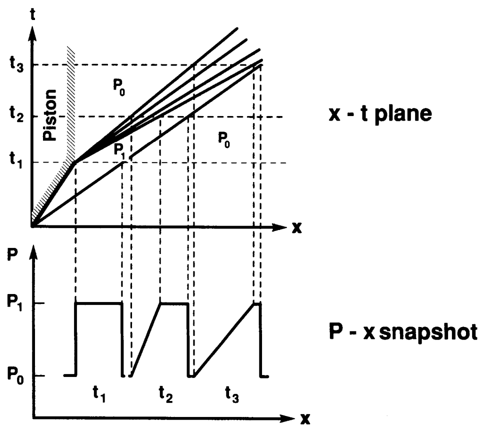

Figure 2.8. An x–t diagram of a piston interacting with a compressible fluid. At the origin, the piston begins moving at constant velocity, generating a shock wave. At t_1, the piston stops abruptly, generating rarefaction fan. "Snapshots" of wave profiles at times t_2 and t_3 are shown.

a rate-independent constitutive relation, the abscissa could just as easily have been shock velocity, particle velocity, internal energy, or strain. Such a pulse could be generated in the tube depicted in Fig. 2.3 by instantaneously accelerating the piston from rest at $t = 0$ to constant velocity for some time interval Δt, and then returning it to rest. If the wave is propagating in a material that supports shock waves, the leading edge of the pulse remains steep and moves at the shock velocity, while the trailing edge spreads with time. The interval of time that the pressure resides at its peak value decreases because the leading part of the rarefaction wave moves faster than the shock, according to the stability conditions (2.31). Eventually, the shock front is overtaken by the rarefaction and begins to decay.

The most convenient way to represent this information graphically is by way of an "x–t diagram" (Fig. 2.8), where trajectories of shocks, rarefactions, and interfaces are plotted in the x–t plane. In this case, the origin is the initial position of the piston–fluid interface at the instant it starts to move at constant velocity u. There are two lines that radiate from this point: one corresponds to the interface, with slope $1/u$, and the other represents the faster shock wave, with slope $1/U$. At time Δt, the piston stops, so the line representing it becomes vertical. At this point the expansion wave is generated, but because it spreads out it cannot be drawn as a single line. Its leading edge, or "rarefaction front", is faster than the shock wave, so its slope is shallower and the two lines eventually intersect. The trailing edge, or "rarefaction tail", is slower, so it spreads away from the rarefaction front in a "rarefaction fan." By drawing horizontal "isochrons" on the x–t diagram, points on these lines can be correlated with various features of the P–x snapshots at different times.

Another way of representing shock-wave profiles is in the form of "P–t histories" of the pressure or another variable at a series of points along its direction of propagation, as in Fig. 2.9. In the above example, the leading part of the shock front arrives first, effectively increasing the pressure instantaneously. The rarefaction arrives later and decreases the pressure over a time

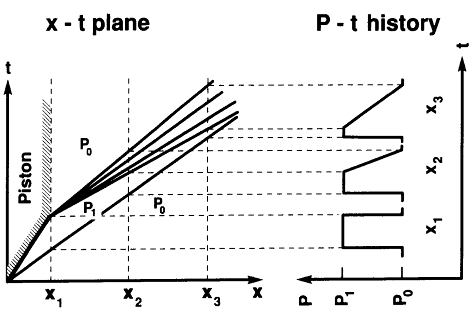

Figure 2.9. The same x–t diagram as in the previous figure is used to illustrate "histories" of material as it flows past points x_1, x_2, and x_3.

interval that increases with propagation distance. In this case, the P–t histories are read from the x–t diagram by drawing vertical lines at several positions of interest.

For simplicity, we have shown an expansion wave in which the pressure is linearly decreasing with time. This, in general, is not the case. The release behavior depends on the equation of state of the material, and its structure can be quite complicated. There are even conditions under which a "rarefaction shock" can form (see Problems, Section 2.20; Barker and Hollenbach, 1970). In practice, there are many circumstances where the expansion wave does not propagate far enough to fan out significantly, and can be drawn as a single line in the x–t diagram.

The P–t histories illustrated by Fig. 2.9 are not histories of a particle of material moving with the flow, because the coordinate that is fixed is x, and material is flowing past it. A more useful P–t history would use a coordinate system which is attached to the material itself, as a stress or particle velocity gauge would be. Such a coordinate system is defined in the next section.

2.10. Eulerian and Lagrangian Coordinates

Thus far, our discussions have considered an "Eulerian" coordinate system in the laboratory frame of reference. This frame of reference is fixed in space, and particles of material pass through it when they are in motion. The Eulerian coordinates are sometimes chosen in a frame of reference in which the velocity of unshocked material is zero, but that is not required. It is also referred to as the "spatial" coordinate system, or "laboratory" coordinate system when the frame of reference is chosen such that the observer is stationary. For example, a useful coordinate system in which to make measurements of the event depicted in Fig. 2.2 would be to number the railroad ties. The railroad tie over which a part of the train is passing would be the Eulerian coordinate of that car. In this particular Eulerian frame, it is the shocked material that has zero velocity. The x–t diagrams of the previous section made use of Eulerian coordinates. Figure 2.10(a) depicts an Eulerian x–t diagram of a shock wave passing through an initially moving medium.

While the Eulerian system has intuitive appeal, it is the "Lagrangian" coordinate system that is more convenient mathematically and in many practical applications. In this system, the coordinate is fixed to the material and moves with it. It is sometimes called the "material" coordinate system. In Fig. 2.2, the boxcars can be numbered, so the position of a car in this system never changes. By convention, the Lagrangian coordinate (h) is chosen so that it is equal to the Eulerian coordinate (x) at some time $t = 0$. Figure 2.10(b) illustrates a Lagrangian h–t diagram of the same system as shown in Fig. 2.10(a) with the Eulerian system. Because the flow is independent of the coordinate system chosen to describe it, both systems must lead to the same results.

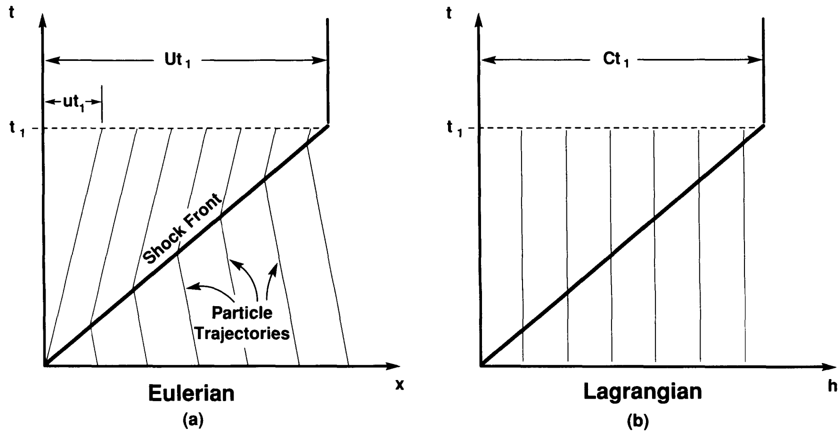

Figure 2.10. (a) An Eulerian x–t diagram of a shock wave propagating into a material in motion. The fluid particle travels a distance ut_1, and the shock travels a distance Ut_1 in time t_1. (b) A Lagrangian h–t diagram of the same sequence. The shock travels a distance Ct_1 in this system.

Part of the utility of Lagrangian coordinates is realized when considering the shock velocity. When viewed in Eulerian coordinates, its value depends on both the particle velocity of the medium into which it is advancing and on the amount of deformation the medium has already experienced. But when measured in Lagrangian coordinates its value depends only on the strength of the shock wave and the Hugoniot of the material. To illustrate this point, consider a shock wave passing through a slab of material with an original thickness of Δx_0 (see Fig. 2.11). If the slab has already been shocked to a uniform state, it has been compressed to the specific volume V_1, so its new thickness is $\Delta x_1 = (V_1/V_0)\,\Delta x_0$, and its particle velocity will be u_1. In the Eulerian system to which we are accustomed, the transit time for the second shock wave through the slab is

$$\Delta t = \Delta x_0(V_1/V_0)/(U - u_1). \tag{2.38}$$

On the other hand, the thickness of the slab in the Lagrangian system is the constant $\Delta h_0 \equiv \Delta x_0$, so for a Lagrangian shock velocity of C, the transit time is

$$\Delta t = \Delta x_0/C. \tag{2.39}$$

The transit time must be independent of the coordinate system, so these expressions can be equated, yielding

$$C = (V_0/V_1)(U - u_1). \tag{2.40}$$

The relative shock velocity $U' \equiv U - u_1$ is the Eulerian shock velocity often used because it is a material property and is independent of the motion of the

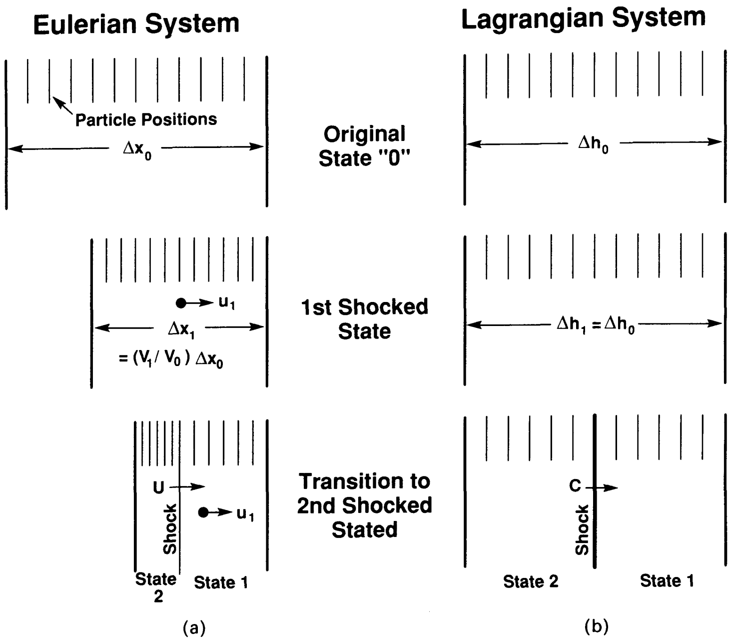

Figure 2.11. The sequence described in the text for (a) Eulerian and (b) Lagrangian coordinates. "Hashmarks" indicate material particle positions.

medium.[4] It is related to the Lagrangian shock velocity by the equation

$$C = (V_0/V_1)U'. \tag{2.41}$$

The form of the jump conditions also depends on the coordinate system. Substituting (2.40) into the general Eulerian form of the momentum jump condition (Table 2.1) yields the Lagrangian jump condition

$$P - P_1 = \rho_0 C(u - u_1). \tag{2.42}$$

2.11. Flow Equations in One Dimension

The introduction of Lagrangian coordinates in the previous section allows a more natural treatment of a continuous flow in one dimension. The derivation of the jump conditions in Section 2.2 made use of a mathematical discontinuity as a simplifying assumption. While this simplification is very useful for many applications, shock waves in reality are not idealized mathematical

[4] The term "shock velocity" usually refers to this Eulerian shock velocity relative to the particle velocity of the unshocked material unless otherwise stated, and is usually written as "U" (without the prime).

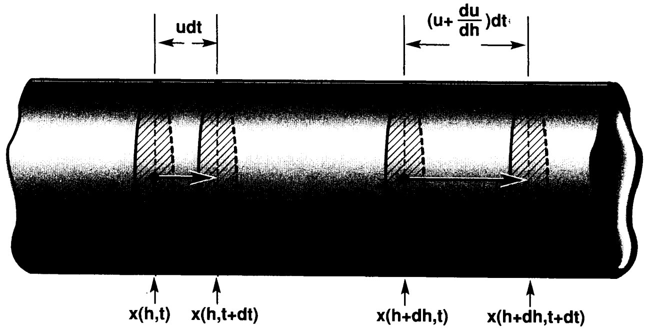

Figure 2.12. A flow tube used to derive one-dimensional flow equations in Lagrangian coordinates. Internal surfaces are massless, impermeable partitions to aid in visualizing elements of fluid in Lagrangian coordinates.

discontinuities; it is important to develop the ability to describe them in terms of continuous flow.

We again consider a fluid-filled flow tube in which the medium can only move in the x direction. The Eulerian coordinate (x) can be envisioned as a ruler fixed to the wall of the tube. The Lagrangian coordinate (h) can be thought of as a series of impermeable, massless walls that separate elements of fluid and are carried along with the flow. The value of a Lagrange coordinate is assigned to each imaginary partition. If two adjacent walls are labeled with Lagrangian coordinates "h" and "$h + dh$," the flow of the fluid element between them can be considered (Fig. 2.12). At time t, the Eulerian coordinates of each side of the fluid element are $x(h, t)$ and $x(h + dh, t)$. The diameter of the flow tube can be chosen so that the fluid element has unit volume at time $t = 0$. Its mass is $\rho_0 \, \Delta h$, where ρ_0 is the density of the fluid in some reference state ($t = 0$).

Since the Lagrangian walls are impermeable, the mass of the Lagrangian element is constant. At time t, when the walls of the element are separated by an Eulerian distance dx, the density of the fluid within it must be

$$\rho = \rho_0 \, dh/dx \qquad \text{(at time } t\text{)}. \tag{2.43}$$

Thus an expression of conservation of mass is

$$\rho_0/\rho = (\partial x/\partial h)_t. \tag{2.44}$$

This equation can be put in a more useful form by differentiating with respect to time and rearranging

$$(\rho_0/\rho^2)(\partial \rho/\partial t)_h + (\partial u/\partial h)_t = 0 \qquad \text{(mass conservation)}, \tag{2.45}$$

where we have used the chain rule and the definition of particle velocity, $u \equiv (\partial x/\partial t)_h$.

The compressive force acting per unit area on the mass element from the left-hand side is P, the pressure at coordinate h. On the right-hand side of the element, this force is equal to $P + (\partial P/\partial h)_t\, dh$, so the net force acting from left to right is $-(\partial P/\partial h)_t\, dh$. Setting this force equal to the product of the mass, $\rho_0\, dh$, and the acceleration, $\partial^2 x/\partial t^2 = \partial u/\partial t$, yields the equation for conservation of momentum

$$(\partial u/\partial t)_h + (\partial P/\partial h)_t/\rho_0 = 0 \qquad \text{(momentum conservation).} \qquad (2.46)$$

Over the time increment dt, the force applied on the left-hand side of the element acts over a distance $u\, dt$, so the work done on the element from the left is $Pu\, dt$. The force on the right-hand boundary of the element is $P + (\partial P/\partial h)_t\, dh$, and it travels a distance $(u + (\partial u/\partial h)_t\, dh)\, dt$, so the work done by the element on the surrounding fluid to the right is $(P + (\partial P/\partial h)_t\, dh)(u + (\partial u/\partial h)_t\, dh)\, dt$. The net work done on the fluid element is the difference

$$\begin{aligned} dW &= Pu\, dt - (P + (\partial P/\partial h)_t\, dh)(u + (\partial u/\partial h)_t\, dh)\, dt \\ &= -((P(\partial u/\partial h)_t\, dh) + u(\partial P/\partial h)_t\, dh))\, dt \\ &= -(\partial(uP)/\partial h)\, dh\, dt. \end{aligned} \qquad (2.47)$$

The time rate of change of total specific energy of the element due to work is thus $(\partial(uP)/\partial h)/\rho_0$. If heat is added to the element from the surroundings at the rate $(\partial Q/\partial t)_h$ per unit mass, then these contributions from work and heat can be equated to the change in internal and kinetic energy per unit mass

$$(-\partial(uP)/\partial h)/\rho_0 + (\partial Q/\partial t)_h = \partial(E + \tfrac{1}{2}(u^2))/\partial t. \qquad (2.48)$$

Making a substitution from (2.46) yields

$$(\partial E/\partial t)_h = (\partial Q/\partial t)_h - (P/\rho_0)(\partial u/\partial h)_t \qquad \text{(energy conservation).} \qquad (2.49)$$

Equations (2.45), (2.46), and (2.49) express the conservation of mass, momentum, and energy in Lagrangian coordinates for continuous flow.

2.12. *P–u* Diagrams

We will be concerned with the interaction of waves with boundaries and with other waves throughout this text. To determine how these interactions take place, it is important to consider that discontinuities in either pressure or particle velocity cannot be sustained in any material. If a discontinuity in either of these variables is created at some point by impact or wave interaction, the resulting motion will be such that the pressure and particle velocity become continuous across the boundary or point of interaction. Unless the material separates at that point, the motion will consist of one or more waves propagating away from the point of the discontinuity. For pressure discontinuities, it is easy to see that waves must propagate by again considering an imaginary flow tube. Whenever the pressures are different at the ends of the tube, there is a net force on the material within it, and it must be accelerated. When there is a discontinuity in particle velocity, lack of wave propagation would either result in interpenetration or separation of the material. The first is a physical impossibility, while the second removes the discontinuity without the requirement of further wave motion. It is useful to note here that other variables, such as density, can be discontinuous without giving rise to wave motion. Such a discontinuity is known as a "*contact discontinuity*" (see Section 2.15).

Because of this special property of pressure and particle velocity, it is the representation of the Hugoniot in the P–u plane that is used to consider wave interactions. Because the Hugoniot is a material property, the relation between P and u cannot depend either on direction of wave motion or frame of reference. The Hugoniot can be freely translated and reflected in the P–u plane to account for initial particle velocity and direction of shock propagation. Thus, a shock wave propagating to the left in an initially stationary body will have a Hugoniot that increases to the left, and be centered at the origin. The Hugoniot corresponding to a shock wave moving into a body in motion at velocity u_0 will be centered at $P = 0, u = u_0$, and increase to the left or right depending on the direction of the wave.

Wave interactions will often cause a solid to be shocked more than once. For materials initially at standard conditions, the state achieved by the first shock wave must lie on the principal Hugoniot. Any subsequent shock wave is centered on that preshocked state, and in general will lie on a different "recentered," or "second" Hugoniot. However, for small strains in most materials it can be shown that the difference between the principal and second Hugoniots is negligible for most purposes (see Problems, Section 2.20).

In Section 2.8, we noted that most expansion waves are isentropic. It was shown in Section 2.4 that the difference between the Hugoniot and isentrope is small for hydrodynamic materials at small strains. Thus we can also represent relief waves in the P–u plane with the same curve used to represent shock waves, if the strains are not too large.

It is convenient to define the shock *impedance* $Z \equiv \rho_0 U$. The shock impedance is a measure of the ability of a material to generate pressure under given loading conditions. According to (2.28), Z is equal to the square root of the Rayleigh line in the P–V plane. The following sections will help develop the concept by considering shock-wave interactions between materials of varying shock impedances. For materials that have Hugoniots that are everywhere upwardly concave in the P–V plane, it is clear that the shock impedance increases with increasing shock pressure, for a single shock from the initial state. In the limit of small shocks, the Hugoniot can be approximated by a straight line. Thus, for an acoustic wave, the impedance is considered to be independent of pressure; the material has a "linear" response, and is described by the constant "acoustic impedance."

2.13. Surface–Surface Interactions

For interactions in which material at a given point experiences only one shock wave and no other shocks or rarefactions, the principal Hugoniot is all that is required to determine the states achieved. In situations in which a material experiences several shocks or rarefactions, using the principal Hugoniot to approximate subsequently shocked or released states may or may not be a good approximation. In this section, only singly-shocked materials at small strains are considered.

EXAMPLE 1 (The Rigid-Piston Problem). Consider the situation depicted in Fig. 2.13. At time $t = 0$, a piston in contact with a compressible fluid is accelerated to constant velocity. A shock wave is propagated from left to right at velocity U, and the material moves with particle velocity u_1, the same as the piston velocity. This interaction is illustrated more simply by the x–t and P–u diagrams in Fig. 2.13. The initial position of the piston–fluid interface is defined as $x = 0$. For $t > 0$, the interface is moving at velocity u_1, so the interface has a slope $= 1/u_1$ in the x–t plane. The shock wave generated from this same point in the x–t plane has slope $1/U$. The state of the fluid before it is disturbed is given the label "0," and is at rest ($u = 0$) and is not under stress ($P = 0$). This label applies to an entire region in the x–t plane (vertical shading) but only to a single point in the P–u plane (the origin). After the shock wave passes, the fluid must be in a state on its Hugoniot, and it must have the same particle velocity (u_1) as the piston. These conditions are satis-

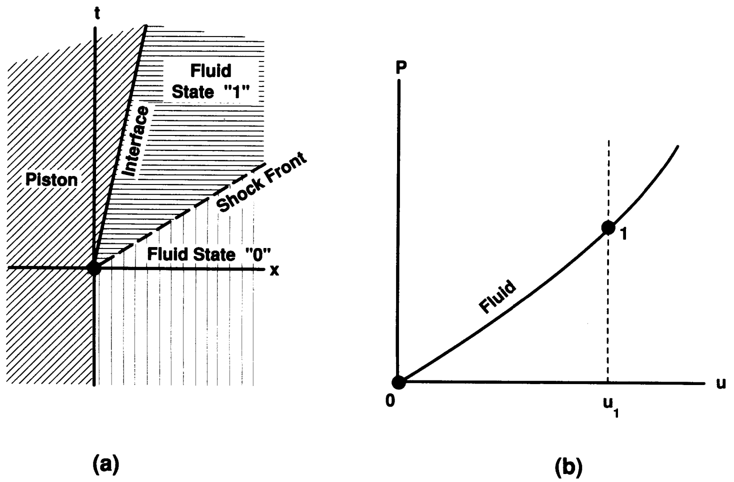

Figure 2.13. (a) x–t and (b) P–u diagrams for the rigid-piston problem. State "0" is at the origin of the P–u plane, state "1" must be on the Hugoniot of fluid with the particle velocity determined by piston velocity.

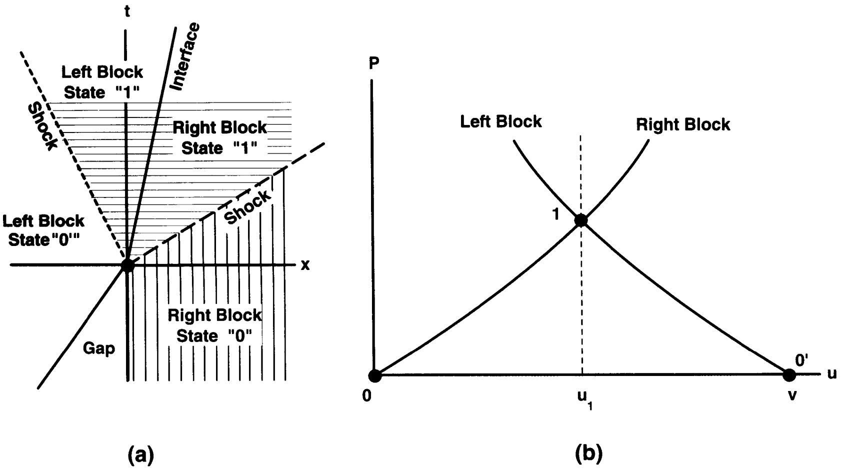

Figure 2.14. (a) x–t and (b) P–u diagrams for symmetric impact. Shock state "1" must be on the Hugoniot of both blocks.

fied at point "1" in the P–u plane; so state "1" applies to the entire region of the x–t plane corresponding to shocked fluid (horizontally shaded).

EXAMPLE 2 (Symmetric Impact). Now suppose two blocks of an identical material impact with a relative velocity v. We can define a frame of reference in which the block on the right-hand side is initially stationary. The x–t and P–u diagrams for this collision are depicted in Fig. 2.14. The time of impact is defined as $t = 0$. The upper right-hand quadrant of the x–t diagram is identical to that for the previous example, if the right-hand block is identified with the fluid and the left-hand block is acting as the piston. The difference is that the particle velocity associated with the shocked material is no longer equal to the impact velocity; this time a shock wave is also driven backward into the "piston" block which is made of the same compressible fluid as the other block. As before, the right-hand material is initially at state "0," with $u = 0$ and $P = 0$. The impacting material is initially in state "0′," with $u = v$ and $P = 0$. The material behind the two shock fronts must lie on both Hugoniots, and have the same pressure and particle velocity. Because the material in the left-hand block is decelerated to a lower particle velocity, its Hugoniot is reflected. These Hugoniots intersect at state "1," which by symmetry has a particle velocity of $u_1 = v/2$.

EXAMPLE 3 (Asymmetric Impact). If the impacting block has a higher shock impedance than the target block, the impact is asymmetric. The P–u diagram for this case is shown schematically in Fig. 2.15. The impactor Hugoniot is steeper in the P–u plane than the target Hugoniot. For an impact velocity of v, it is clear that the shock pressure is higher than the previous case. If the

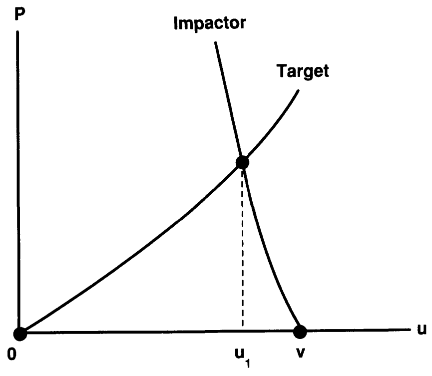

Figure 2.15. P–u diagram for asymmetric impact.

shock impedance of the impactor is taken to infinity, this case is equivalent to the rigid piston problem of Example 1, where $v = u_1$.

2.14. Wave–Surface Interactions

The interactions discussed in this section involve rarefactions as well as shock waves. Provided that strains are small, the release path can be approximated by the Hugoniot in the P–u plane. The following graphical solutions to the interactions are approximate, but in many cases the approximation is very good.

EXAMPLE 1 (Shock Reflection from a Free Surface (Free Surface Approximation)). Consider a shock wave propagating from left to right in a material. The shock wave takes the material from state "0" to state "1" as depicted in Fig. 2.16. When the shock wave reaches a free surface (at the t axis in Fig. 2.16(a)), the free surface boundary condition requires that the stress returns to zero. A rarefaction wave is reflected back into the material, releasing the pressure and accelerating material to the right. For a left-going expansion wave, the release path is approximated by a reflected Hugoniot that intersects the principal Hugoniot at state "1." The free surface boundary condition ($P = 0$) is satisfied on this curve only for state "2," where $u = 2u_1$. The free surface velocity is often measured experimentally, and, making use of this "*free surface approximation*," is used to determine the particle velocity of the first shock state. The free surface approximation is equivalent to the small strain approximation that allows us to replace the release isentrope with the principal Hugoniot.

EXAMPLE 2 (Shock Reflection from a Lower Z Interface). Figure 2.17 represents a shock wave propagation from left to right in material "A." When the

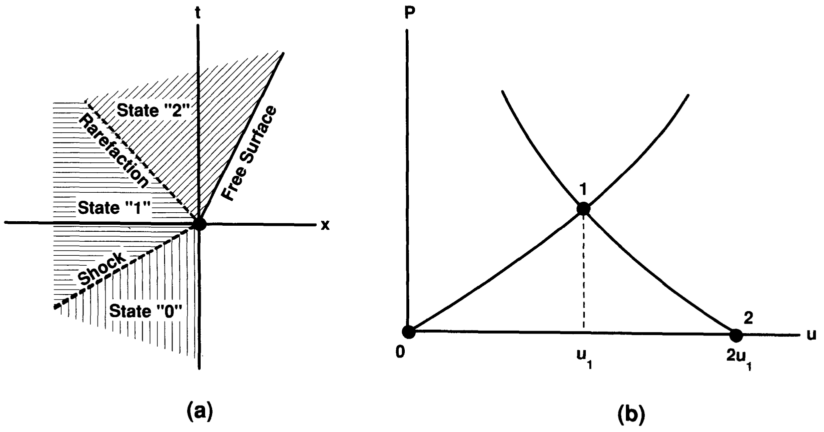

Figure 2.16. (a) x–t and (b) P–u diagrams for shock reflection from a free surface.

shock wave reaches the origin of the x–t diagram, it impinges on an interface with material "B," which has a lower shock impedance, as described by the Hugoniot shown in the P–u diagram. In this case, the free surface boundary condition is replaced by the interfacial boundary condition described in Section 2.12. This condition requires that the waves generated by the interaction lead to a P–u state that is the same on both sides of the interface. For material "A," that state must be reached from state "1" by a single transition between points on a reflected Hugoniot of material "A" (in the small strain approxima-

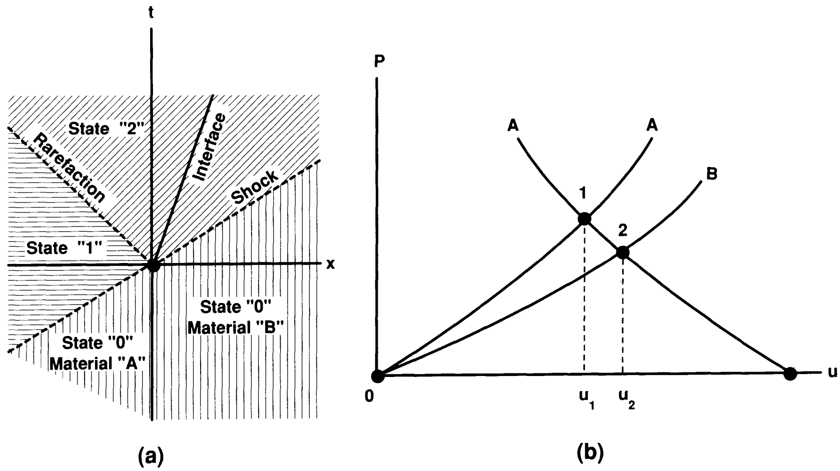

Figure 2.17. (a) x–t and (b) P–u diagrams for shock reflection from the lower Z interface.

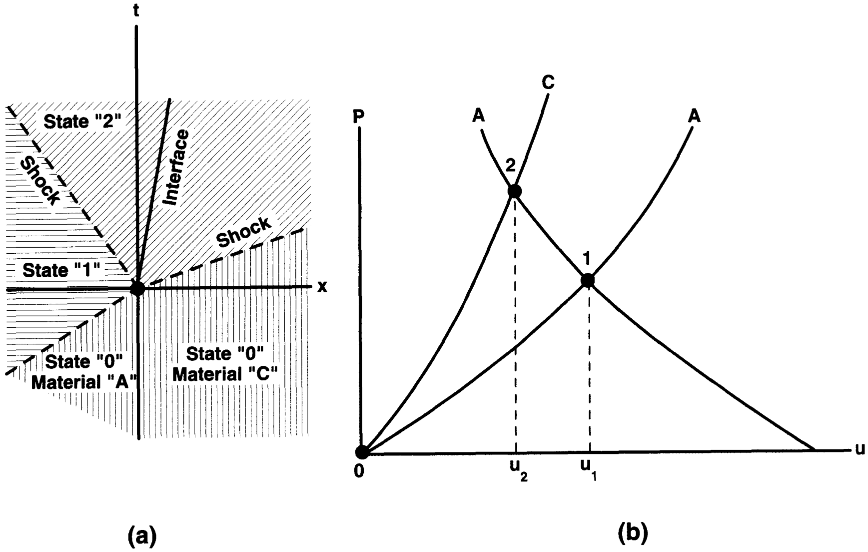

Figure 2.18. (a) x–t and (b) P–u diagrams for shock reflection from the higher Z interface.

tion). For material "B," the transition must be made between points on the material "B" Hugoniot, where the first point is state "0." The only common P–u state is the point of intersection of these two Hugoniots; state "2." It should be clear from Fig. 2.17 that the interaction results in a reflected release wave in material "A" and a transmitted shock (of lower amplitude) in material "B."

Example 3 (Shock Reflection from a Higher Z Interface). If material "B" is replaced by material "C," with a higher shock impedance than "A," the same interfacial boundary conditions can be applied. In this case, however (Fig. 2.18), this resulting state ("2"), is at higher pressure than state "1." The wave reflected back into material "A" is a shock. If the right-hand block of material is replaced by another block of material "A," there are no reflected waves, and the transmitted shock is identical to the incident wave.

2.15. Wave–Wave Interactions

Example 1 (Shock–Shock Collision). An interaction consisting of a head-on collision between two shock waves is illustrated by Fig. 2.19. If a right-going shock wave collides with a left-going shock of different amplitude (in this case, higher), the (P, u) boundary condition must be applied to the point (or interface) of collision; waves must be propagated such that P and u become the same on both sides of this interface. In this case, the material on both sides of

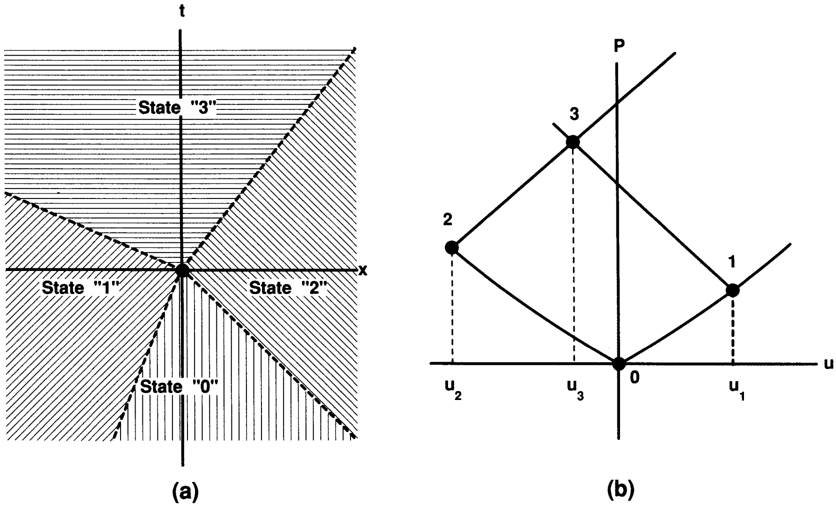

Figure 2.19. (a) x–t and (b) P–u diagrams for shock–shock collision.

the interface is the same and has the same Hugoniot. The resulting left-going wave must take the material to a state lying on a reflected Hugoniot passing through state "1." Likewise, the resulting right-going wave reaches the same P, u state, lying on a translated Hugoniot passing through state "2." These two curves intersect at state "3," the final state.

It is important to note that the "state" determined by this analysis refers only to the pressure (or normal stress) and particle velocity. The material on either side of the point at which the shock waves collide reach the same pressure–particle velocity state, but other variables may be different from one side to the other. The material on the left-hand side experienced a different loading history than that on the right-hand side. In this example the material on the left-hand side would have a lower final temperature, because the first shock wave was smaller. Such a discontinuity of a variable, other than P or u that arises from a shock wave interaction within a material, is called a "*contact discontinuity*." Contact discontinuities are frequently encountered in the context of inelastic behavior, which will be discussed in Chapter 5.

Example 2 (Rarefaction–Rarefaction Interaction). Suppose a slab of material with two free surfaces is initially at some state of high pressure. Because the free surface boundary conditions must apply on both faces of the slab, rarefaction waves must propagate inward, as depicted by Fig. 2.20. If the slab is initially stationary ($u = 0$), the right-hand side is accelerated to the right and vice versa. At some plane internal to the slab, the rarefactions meet. At the instant of interaction, the material on the right of this interface is moving to the right with particle velocity u_1 and the material on the left is moving to

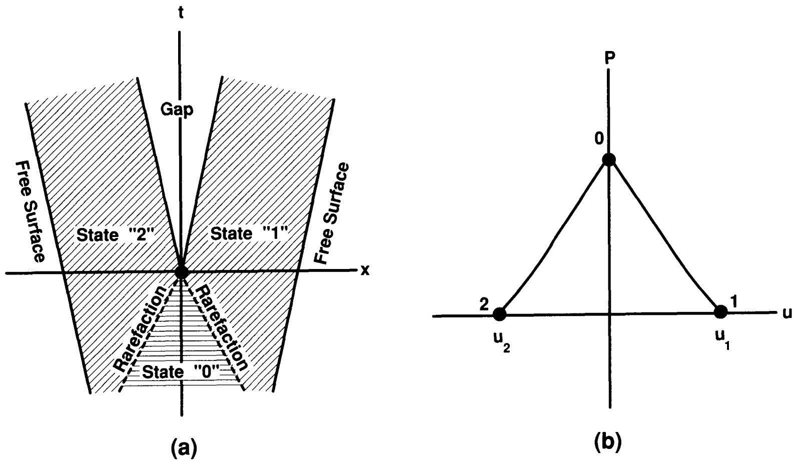

Figure 2.20. (a) x–t and (b) P–u diagrams for release–release collision.

the left at velocity $u_2 = -u_1$. For a zero-strength material, the slab separates at this plane, each side continuing in its respective direction at its particle velocity.

For an actual material with strength, the situation is much more complicated. One problem is that release waves are not discontinuities; in reality, they spread out as they propagate. Another complication is that materials with strength can support tension, so the release curves in the P–u plane can be extended to "negative pressure." The magnitude of tension that can be sustained by a material depends not only on its strength, but can depend on its past loading history. When the tension exceeds the strengh, the material splits apart, or "*spalls*." Such considerations are beyond the scope of this chapter, and will be addressed in Chapter 3.

2.16. Entropic Effects

We have previously stated that, for small strains, the difference between the principal Hugoniot and the isentrope centered at ambient conditions is small. This can be shown by expanding the entropy S in a Taylor series along the Hugoniot

$$S = S_0 + \left.\frac{dS}{dV}\right|_0 (V - V_0) + \frac{1}{2!}\left.\frac{d^2S}{dV^2}\right|_0 (V - V_0)^2 + \frac{1}{3!}\left.\frac{d^3S}{dV^3}\right|_0 (V - V_0)^3 + \cdots . \tag{2.50}$$

By taking the total differential of the Rankine–Hugoniot equation (2.4)

$$dE = \tfrac{1}{2}(V_0 - V)\,dP - \tfrac{1}{2}(P + P_0)\,dV, \tag{2.51}$$

and combining with the differential expression of the first and second laws of thermodynamics

$$dE = T\,dS - P\,dV, \tag{2.52}$$

the first-order derivative in the Taylor expansion can be written

$$\left.\frac{dS}{dV}\right|_0 = \frac{1}{2T}\left[P - P_0 + (V_0 - V)\frac{dP}{dV}\right]_{P=P_0,\,V=V_0} = 0. \tag{2.53}$$

Similarly, the second-order derivative can be shown to be zero (see, Problems, Section 2.20). Evaluating the third term gives an expression for entropy gain along the Hugoniot

$$S - S_0 = \frac{1}{12T_0}\left.\frac{d^2P}{dV^2}\right|_0 (V_0 - V)^3 + O(V_0 - V)^4. \tag{2.54}$$

Thus, to third order in strain, the entropy along the Hugoniot is constant, and weak shock waves are nearly isentropic. For small strains, the Hugoniot can be replaced with the isentrope to a high degree of accuracy. At the initial state, the Hugoniot and isentrope have the same slope and curvature in the P–V plane.

As pointed out in Section 2.4, shock waves are such rapid processes that there is no time for heat to flow into the system from the surroundings; they are considered to be adiabatic. By the second law of thermodynamics, the quantity $(S - S_0)$ must be positive for any thermodynamic process in an isolated system. According to (2.54), this quantity can only be positive if the P–V isentrope is concave upward. Thus, the thermodynamic stability condition for a shock wave is

$$\left.\frac{d^2P}{dV^2}\right|_s > 0. \tag{2.1}$$

By approximating the isentrope by the Hugoniot, the amount of irreversible heating experienced by a material that undergoes a shock/rarefaction cycle can be visualized. In Fig. 2.5, the internal energy increase is the trapezoidal area underneath the Rayleigh line. During release, the thermodynamic path is (usually) isentropic, so the work returned to the surroundings is approximately the area under the Hugoniot. The internal energy retained on release is approximately equal to the crescent-shaped area between the Rayleigh line and the Hugoniot. This internal energy is that due only to the irreversible nature of the shock wave and the entropy gained, and is called the "*waste heat.*"

2.17. Riemann Integral

According to (2.29), the change in particle velocity across a discontinuity is $u - u_0 = \sqrt{(P - P_0)(V_0 - V)}$. The incremental change can be written as a differential

$$du = \pm\sqrt{-dP\,dV}, \tag{2.55}$$

where the sign depends on the direction of flow. Because an unsteady wave, such as a spreading rarefaction, can be approximated by an infinite number of vanishingly small discontinuities, equation (2.55) can be applied. According to (2.54), the entropy gain is zero in the limit of small shocks, so we can set the derivative dP/dV equal to the isentropic partial derivative $(\partial P/\partial V)_s$, and write

$$du = \pm dP\sqrt{1/(\partial P/\partial V)_s} = \pm dP/\rho a, \tag{2.56}$$

where the definition of the adiabatic bulk sound speed

$$a = (1/\rho)\sqrt{(\partial P/\partial V)_s} \tag{2.57}$$

has been used.

Equation (2.56) can be integrated to provide a means of determining the relationship between pressure and particle velocity for continuous flow

$$u - u_0 = \int_{P_0}^{P} (dP/\rho a). \tag{2.58}$$

This result, called the "Riemann Integral," can be applied to unsteady isentropic compression waves as well as to expansion waves. By defining a "Riemann function"

$$L(P) = \int_{0}^{P} (dP/\rho a), \tag{2.59}$$

the equation (2.58) can be rewritten as

$$u + L(P) = u_0 + L(P_0) = \text{constant}. \tag{2.60}$$

Note that in arriving at (2.58) it was assumed that there was no change in entropy along the rarefaction. This assumption is equivalent to stating that the rarefaction must be propagating into a uniform state. The "Riemann Invarient" has been defined in terms of the Riemann function

$$s = \tfrac{1}{2}(L - u). \tag{2.61}$$

This constant is independent of position on the rarefaction wave.

2.18. Summary

In this chapter we introduced the concept of shock waves, ignoring the features that distinguish solids from fluids. The properties include shear strength, polymorphic phase transformations, heterogeneous structure, anisotropy, and viscoplastic behavior. These topics make up the majority of the subject of shock compression of solids, and form a large portion of the rest of this book.

To lay the framework for shock compression, we showed that mass, momentum, and energy are conserved across a shock discontinuity for flow in one dimension. A simple model involving beads on a wire was introduced to help visualize the shock wave concept. By considering thermodynamics and the compressive properties of condensed matter, the Hugoniot curve was defined as the locus of possible states achievable by a single shock transition, and a material property. A differential equation for the Hugoniot curve was derived and written in terms of compressive and thermal properties of materials. As an aid to visualizing shock-wave concepts, certain quantities associated with shock waves, such as shock velocity, internal energy, and kinetic energy were related to graphs of shock-wave variables.

The properties required of a material in order for it to support a stable shock wave were listed and discussed. Rarefaction, or release waves were defined and their behavior was described. The useful tool of plotting shocks, rarefactions, and boundaries in the time–distance plane (the "x–t" diagram) was introduced. The Lagrangian coordinate system was defined and contrasted to the more familiar Eulerian coordinate system. The Lagrangian system was then used to derive conservation equations for continuous flow in one dimension.

By plotting Hugoniot curves in the pressure–particle velocity plane (P–u diagrams), a number of interactions between surfaces, shocks, and rarefactions were solved graphically. Also, the equation for entropy on the Hugoniot was expanded in terms of specific volume to show that the Hugoniot and isentrope for a material is the same in the limit of small strains. Finally, the Riemann function was derived and used to define the Riemann Invarient.

2.19. Acknowledgments

We express our gratitude to Orval E. Jones, George E. Duvall, and Dennis B. Hayes whose unpublished notes have instructed a generation of shock-wave scientists and engineers at Sandia and elsewhere. This excellent source of information figures prominently in this chapter. The figures were skillfully drawn by Kay Lang.

2.20. Problems

2.1. Determine the P–V Hugoniot of a material whose equation of state is given by

$$E(P, V) = (P + K)V/\Gamma,$$

where Γ and K are constants. Express P as a function of $\varepsilon = (1 - V/V_0)$.

2.2. Derive (2.28) and (2.29) from the jump conditions.

2.3. An interesting result occurs when the beads on a wire stick together upon contact. In this case, an additional energy resides at the interface between particles. Determine the jump conditions for this case as was done for perfectly elastic beads, and draw an x–t diagram.

2.4. Determine the conditions for stability of a rarefaction shock.

2.5. Show that, for small strains, the difference between the principal and the second Hugoniot of a material is negligible.

2.6. What happens when a shock wave collides head-on with a release wave?

2.7. Show that the third term in (2.50) is zero.

2.8. Derive the expression for entropy on the Hugoniot (2.54). Relate to thermal quantities γ, T, and C_v.

2.9. Consider the impact of a semi-infinite space on a plate of thickness dp, separated from an identical plate by a gap of width d_g. If the impactor and plates are all composed of the same materials, what is the subsequent behavior? Plot in both Lagrangian and Eulerian coordinates.

2.10. Show whether or not a shock wave is stable in a material with a linear Hugoniot in the P–V plane.

2.11. Write the Eulerian sound speed, a, in terms of the Lagrangian sound speed, c.

2.21. Glossary

Adiabatic: A process for which there is no heat transfer between a system and its surroundings. An adiabatic process that is reversible is *isentropic*.

Contact discontinuity: A spatial discontinuity in one of the dependent variables other than normal stress (or pressure) and particle velocity. Examples such as density, specific internal energy, or temperature are possible. The contact discontinuity may arise because material on either side of it has experienced a different loading history. It does not give rise to further wave motion.

Conservation equations: Expressions that equate the mass, momentum, and energy across a *steady wave* or shock discontinuity ((2.1)–(2.3)). Also known as the *jump conditions* or the *Rankine–Hugoniot relations*.

Constitutive relation: An equation that relates the initial state to the final state of a material undergoing shock compression. This equation is a property of the material and distinguishes one material from another. In general it can be rate-dependent. It is combined with the *jump conditions* to yield the *Hugoniot curve* which is also material-dependent. The *equation of state* of a material is a constitutive equation for which the initial and final states are in thermodynamic equilibrium, and there are no rate-dependent variables.

Density: The mass per unit volume of a material. The reciprocal of *specific volume*.

Eulerian coordinates: The coordinate system in which the spatial position (x) and time (t) are the independent variables. The dependent variables are expressed as functions of x as material moves through space. Also known as *laboratory coordinates* when the reference frame is that of an observer.

Equation of state: An equation that describes the properties of a given material, and distinguishes one material from another. It defines a surface in thermodynamic variable space on which all equilibrium states lie. In shock-wave applications, the initial and final states are frequently assumed to lie on the *equation of state surface*, and this equation can be combined with the *jump conditions* to define the *Hugoniot curve* which is material specific.

Lagrangian coordinates: The coordinate system in which the material position (h) and time (t) are the independent variables. The dependent variables are described as functions of a particle position within the material which had coordinate $x = h$ at time $t = 0$. Also known as *material coordinates.*

Hugoniot curve: A curve representing all possible final states that can be attained by a single shock wave passing into a given initial state. It may be expressed in terms of any two of the five variables: shock velocity, particle velocity, density (or specific volume), normal stress (or pressure), and specific internal energy. This curve it not the loading path in thermodynamic space.

Impedance

Shock: Defined as $Z = \rho_0 U$. Describes the ability of material to generate pressure under given loading conditions. Generally a function of pressure.

Acoustic impedance: The *shock impedance* in the limit of an infinitesimal disturbance. Independent of pressure.

Isentropic: A reversible *adiabatic* process, in which there is no change in the entropy of the system.

Jump conditions: Expressions for conservation of mass, momentum, and energy across a *steady wave* or shock discontinuity ((2.1)–(2.3)). Also known as the *conservation equations* or the *Rankine–Hugoniot relations.*

Particle velocity: The velocity associated with a point attached to the material as it flows through space.

Rayleigh line: A chord that connects the initial state of a material on its Hugoniot curve to the final state on the curve. Most frequently drawn in the $P–V$ plane.

Riemann Invarient: A constant defined by (2.61), which is independent of position on a rarefaction wave that propagates into a uniform state.

Rarefaction wave: A wave that reduces the normal stress (or pressure) inside a material as it propagates; the mechanism by which a material returns to ambient pressure after being shocked (the state behind the wave is at lower stress than the state in front of it). Also known as unloading, expansion, release, relief, or decompression waves.

Shock velocity: The velocity of the shock wave as it passes through the material. In the limit of an infinitesimally small shock wave it is equal to the bulk sound speed of the material.

Spall strength: The dynamic tensile strength of a material associated with tension that results from the wave interaction of *rarefaction waves*. When the spall strength is exceeded, the material separates, or "spalls."

Specific volume: The volume of a material taken up by a unit mass. The reciprocal of *density*.

Steady wave: A propagating transition region that connects two uniform states of a material. The wave velocities of all parts of the disturbance are the same, so the profile does not change with time, and the assumptions that go into the *jump conditions* are valid.

Unsteady wave: A loading or unloading wave whose profile changes with time. The *jump conditions* cannot be rigorously applied to such a wave.

x–t diagram: A diagram in which trajectories of shock waves, material interfaces, and contact discontinuities can be plotted.

2.22. References

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